Condensation of phonons in an ultracold Bose gas

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We consider the generation of longitudinal phonons in an elongated Bose-condensed gas at zero temperature due to parametric resonance as a result of the modulation of the transverse trap frequency. The nonlinear temporal evolution with account of the phonon-phonon interaction leads self-consistently to the formation of the stationary state with the macroscopic occupation of a single phonon quantum state.

PACS numbers: 03.75.Kk, 03.75.Nt, 67.40.Db

The problem of the Bose-Einstein condensation of excitations is one of most interesting aspects of the condensed matter physics. It is well known that under condition of thermal equilibrium the chemical potential of excitations vanishes and, as a result, their condensate does not form. The only way to overcome this obstacle is a creation of the stationary state with the conserved number of excitations in a non-equilibrium system. In the present paper we describe a mechanism leading to the formation of such state for phonon excitations in the Bose-condensed atomic gases. The mechanism is based on generating phonons with the help of parametric resonance. In the Bose-condensed gases the parametric resonance (PR) is responsible for the effective energy transfer between different branches of excitations and temporal evolution of the system. In particular, the damping of transverse oscillations in a elongated cylindrical trap at zero temperature is due to PR with the production of longitudinal phonons [1]. The nontrivial picture of the damping in these conditions is revealed in the remarkable work of the Paris group [2]. The specific features of this picture is a result of the nonlinear evolution accompanying PR in such system [3].

We analyze the temporal evolution of interacting longitudinal (sound) phonons in an elongated cylindrical parabolic trap subjected to permanent modulation of the transverse trap frequency $\omega_\perp$. This modulation brings about the oscillations of the condensate density and, accordingly, sound velocity. Due to PR this results in the generation of pairs of sound phonons with the opposite momenta and energies close to a half of the modulation frequency $\omega_0/2$. In fact, phonon pairs are produced within the finite energy interval $\omega_0/2 \pm E_0$. The parameter E_0 is connected with the modulation amplitude $|\delta\omega_\perp/\omega_\perp| \ll 1$ by the relation $E_0 \sim \omega_0 \cdot |\delta\omega_\perp/\omega_\perp|$. If E_0 exceeds the damping factor γ for longitudinal phonons, their occupation numbers begin to increase exponentially in time. In this case the phonon-phonon interactions become significant. As a result, the evolution scenario changes drastically. Let us assume that, due to finite value of longitudinal size L, a single phonon level alone lies within the narrow energy interval $2E_0$. In this case the main effect of the phonon-phonon interactions is an effective renormalization of the level. As is shown below, in the course of the self-consistent nonlinear evolution the renormalized level reaches the left or right edge of the parametric resonance interval in which the growth of the phonon occupation numbers ceases. As a result, the stationary state of phonons with the macroscopic occupation numbers in a single quantum state takes place. In the new state the steady periodic space modulation of the longitudinal density of a Bose gas occurs with the amplitude proportional to the number of condensed phonons.

Let us consider the Bose-condensed gas at $T = 0$ in a trap of the cylindrical symmetry with $L \gg R$, where R is the radius of the condensate. Neglecting the edge effects, we can write the general equation for the field operator $\hat{\Psi}(\mathbf{r}, z, t)$ of atoms in the form

$$i\hbar\frac{\partial\hat{\Psi}}{\partial t}=\Big[-\frac{\hbar^2}{2m}\nabla^2_{\mathbf{r}}-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial z^2}+\frac{m\omega^2(t)\mathbf{r}^2}{2}\Big]\hat{\Psi}+U_0\hat{\Psi}^+\hat{\Psi}\hat{\Psi}, \tag{1}$$

where $U_0 = 4\pi\hbar^2 a/m$, a being the s-scattering length. Note, considering a dilute and cold Bose gas with dominant binary collisions we imply as usually that the radius of interaction region $r_0 \ll a$, the gas parameter $na^3 \ll 1$, and the correlation length $\xi > n^{-1/3}$, n is the atomic density. In this case, the replacement of two-body interaction with the effective contact potential is relevant. The trap transverse frequency depends on time as

$$\omega(t) = \omega_\perp(1 + \eta \sin\omega_0 t), \quad \eta \ll 1. \tag{2}$$

According to the results [4], we introduce the spatial scaling parameter $b(t)$ and new variables $\overrightarrow{\rho} = \mathbf{r}/b(t)$, $\tau(t)$. The field operator in terms of new variables can be written as

$$\hat{\Psi} = \frac{\hat{\chi}(\overrightarrow{\rho}, z, \tau)}{b(t)} \cdot e^{i\Phi}; \quad \Phi = \frac{mr^2}{2\hbar}\cdot\frac{\dot{b}}{b}. \tag{3}$$

Substituting these expressions into Eq.(1), we obtain straightforwardly

$$i\hbar\frac{\partial\hat{\chi}}{\partial\tau}=\Big[-\frac{\hbar^2}{2m}\nabla^2_\rho+\frac{m\omega_\perp^2\rho^2}{2}\Big]\hat{\chi}+U_0\hat{\chi}^+\hat{\chi}\hat{\chi}-b^2(t)\frac{\hbar^2}{2m}\frac{\partial^2\hat{\chi}}{\partial z^2}, \tag{4}$$

provided that functions $b(t)$ and $\tau(t)$ satisfy the equations

$$\frac{d^2b}{dt^2} + \omega^2(t)b = \frac{\omega_\perp^2}{b^3}; \quad b^2\frac{d\tau}{dt} = 1. \tag{5}$$

Treating the condensate wave function as independent of z, we find that Eq.(4) expressed in variables ρ, τ describes the evolution at the time-independent frequency $\omega_\perp$. Combining expression (2) with the condition $\omega_0 \ll \omega_\perp$, we find the solution of Eq. (5) in the form

$$b(t) = 1 + b_1(t), \qquad b_1(t) \approx \frac{-\eta}{2} \cdot \sin \omega_0 t. \tag{6}$$

The field operator $\hat{\chi}$ can be represented in the usual form $\hat{\chi} = (\chi_0 + \hat{\chi}') \exp(-i\mu\tau)$, χ_0 being the condensate wave function and μ being the initial chemical potential. In the absence of excitations at $T = 0$ we can omit the last term in Eq. (4) and determine $\chi_0(\rho)$ in the universal form. Considering excited states, we first linearize Eq. (4) in $\hat{\chi}'$. Under condition $\omega_0 \ll \omega_\perp$ only long-wave longitudinal phonons prove to be involved into the evolution of the system. For these phonons, the transverse distribution is close to χ_0 (see, e.g., [5]). This allows us to simplify the equation obtained. Using the second relation in (5) and going over to variable t, we arrive at the following equation

$$i\hbar \frac{\partial \hat{\chi}'}{\partial t} = \frac{G}{b^2(t)} (\hat{\chi}' + \hat{\chi}'^{+}) - \frac{\hbar^2}{2m} \frac{\partial^2 \hat{\chi}'}{\partial z^2}; \;\; G = U_0 \chi_0^2, \tag{7}$$

Involving the symmetry of the problem, we have for the operator $\hat{\chi}'$ in second quantization

$$\hat{\chi}' = \sum_k \chi_k \hat{a}_k; \qquad \chi_k = \frac{e^{ikz}}{\sqrt{L}} \cdot \phi_0(\overrightarrow{\rho}), \tag{8}$$

where $\hat{a}_k$ being the annihilation operator for atoms. At $\omega_0 \ll \omega_\perp$ the function ϕ_0 is actully close to χ_0 for both the quasi-1D and Thomas-Fermi cases. In the expression (8) we keep the ground state alone for the transverse motion assuming that higher states are insignificant in the course of evolution. Substitution of the expression (8) into Eq.(7) gives after the standard averaging over the radial variables

$$i\hbar \frac{d\hat{a}_k}{dt} = \Big(\bar{G} + \frac{\hbar^2 k^2}{2m}\Big) \hat{a}_k + \bar{G} \hat{a}^+_{-k} - \eta \bar{G} \sin \omega_0 t (\hat{a}_k + \hat{a}^+_{-k}) \tag{9}$$

$\bar{G} = \int d^2\rho \phi_0^2(\overrightarrow{\rho}) G(\overrightarrow{\rho})$. Let us rewrite this equation in terms of phonon operators using the usual transformation $\hat{a}_k = u_k \hat{b}_k + v_k \hat{b}^+_{-k}$. When the last term in Eq. (9) is omitted, a set of equations for $\hat{b}_k, \hat{b}^+_{-k}$ determines the well-known Bogoliubov spectrum ω_k (with the replacement $nU_0 \to \bar{G}$). Considering the general nonstationary case, we introduce the substitution $\hat{b}_k = \hat{\tilde{b}}_k \exp(-i\omega_0 t/2)$ and take into account that long times $\omega_0 t \gg 1$ are most interesting for the analysis. Then, within the resonance approximation the equations for $\hat{\tilde{b}}_k, \hat{\tilde{b}}^+_{-k}$ are reduced to the form

$$i \frac{d\hat{\tilde{b}}_k}{dt} = \xi_k \hat{\tilde{b}} + iE_{0k} \hat{\tilde{b}}^+_{-k}; \quad -i \frac{d\hat{\tilde{b}}^+_{-k}}{dt} = \xi_k \hat{\tilde{b}}^+_{-k} - iE_{0k} \hat{\tilde{b}}, \tag{10}$$

$E_{0k} = (\eta \bar{G}/2\hbar)[(u_k + v_k)/u_k]$, $\xi_k = \omega_k - \omega_0/2$. In fact, we consider the evolution of excitations at the background of the coherently oscillating condensate. These excitations can be found as oscillations of classical Bose-field of the condensate (see, e.g., [6]). With this advantage the operators in Eqs. (10) can be replaced with the classical functions. If the solution of Eqs. (10) is represented in the form $\tilde{b}_k, \tilde{b}^+_{-k} \sim \exp(\alpha_{0k} t)$, one finds $\alpha_{0k} = [|E_{0k}|^2 - \xi_k^2]^{1/2}$. This result demonstrates the appearance of PR with an exponential growth of the phonon occupation numbers at $|E_0| > |\xi_k|$, which is induced by the modulation of the transverse trap frequency. Thus the parametric resonance occurs within the narrow range near ω_0 with the width $2E_0$. For sound phonons $|(u_k + v_k)/u_k| \approx \hbar\omega_k/\mu$ and for the parameter E_0, we find

$$E_{0k} \approx \eta\omega_0 = \omega_0 \frac{|\delta\omega_\perp|}{\omega_\perp} \equiv E_0. \tag{11}$$

So far we have disregarded the phonon-phonon interaction. With an exponential growth of the phonon number the interaction begins to play an essential role. The weakness of the phonon-phonon interaction as itself (see below) allows us to take only three- and four-phonon processes into account. Assuming generation of sound phonons alone in the parametric interval $2E_0$, we can use the expressions for $H^{(3)}$ and $H^{(4)}$ obtained within the hydrodynamic approximation, see [7]. The direct analysis shows that the contribution of $H^{(4)}$ is small in comparison with the term calculated in second order of the perturbation theory with respect to $H^{(3)}$ owing to smallness of the gas parameter. The dominant term in the Hamiltonian of three-phonon interactions has the form

$$H^{(3)} = \frac{m}{2} \int d^3x \hat{\mathbf{v}} \delta\hat{n} \hat{\mathbf{v}}. \tag{12}$$

Here $\hat{\mathbf{v}}$, $\delta\hat{n}$ are the operators of velocity and alternating part of the density, respectively. In fact, we consider the interaction of sound phonons that reduces $H^{(3)}$ to the one-dimensional problem. Since each three-phonon vertex implies the momentum conservation law, at least one of three phonons lies beyond the parametric interval and, therefore, has zero occupation number at $T = 0$. This makes possible to reduce the expression obtained in second order in $H^{(3)}$ to the effective Hamiltonian for the four-phonon interaction ($\Delta(k)$ is the Kroneker symbol)

$$H_{eff} = \frac{\hbar A}{2} \sum_{k_1,\dots,k_4} \hat{b}^+_{k_1} \hat{b}^+_{k_2} \hat{b}_{k_3} \hat{b}_{k_4} \Delta(k_1 + k_2 - k_3 - k_4). \tag{13}$$

Here all states lie within the interval of $2E_0$. Using the known expressions for operators $\hat{\mathbf{v}}$, $\delta\hat{n}$ [7] and condition $E_0 \ll \omega_0$, we obtain $A = \hbar\omega_0^2/\mu N$ where N is the total number of particles. In addition, second order in $H^{(3)}$ contains the imaginary part related to real decay processes of phonons, which determine the phonon damping.

Later, we take into account phenomenologically introducing a decrement γ_k. After work [2] one can conclude that the parameter γ_k is small at $T = 0$ for the geometry under consideration. Therefore, it is rather easy to satisfy the conditions when $\alpha_{0k} > \gamma_k$ and the parametric growth of phonon number remains. Now let us write the equation for $\hat{b}_k$ taking H_{eff} into account. Using substitution $\hat{b}_k = \hat{\tilde{b}}_k \exp(-i\omega_0 t/2)$ and going over to the classical Bose field for phonons, we have

$$\begin{aligned} i\frac{d\tilde{b}_k}{dt} =& (-i\gamma_k + \xi_k)\tilde{b}_k + iE_0\tilde{b}^*_{-k} + \\ &+ A\sum_{k_2k_3k_4} \tilde{b}^*_{k_2}\tilde{b}_{k_3}\tilde{b}_{k_4}\Delta(k + k_2 - k_3 - k_4). \end{aligned} \tag{14}$$

By means of the equations for $\tilde{b}_k$ and $\tilde{b}^*_{-k}$ one can directly obtain the equations for the correlators $N_k =< \tilde{b}^+_k \tilde{b}_k >$ and $f_k =< \tilde{b}_k \tilde{b}_{-k} >$. Using conditions $A \ll E_0 \ll \omega_0$, we decouple the arising four-phonon terms within the mean-field approximation. As a result, we arrive at a set of nonlinear equations that describes the self-consistent evolution of interacting phonons in the vicinity of PR

$$\begin{aligned} \frac{dN_k}{dt} =& -2\gamma N_k + E_0(f_k + f^*_k) + iA(\mathcal{P}^* f_k - \mathcal{P} f^*_k); \\ i\frac{df_k}{dt} =& 2(-i\gamma + \xi_k)f_k + 2iE_0N_k + 2A\mathcal{P}N_k + 4A\mathcal{Q}f_k. \end{aligned} \tag{15}$$

Here $\mathcal{Q} = \sum_{k'} N_{k'}$; $\mathcal{P} = \sum_{k'} f_{k'}$. Hereafter the evident relations $\omega_k = \omega_{-k}$, $N_k = N_{-k}$ are used.

In the general case, Eqs. (15) are a set of nonlinear integral equations. To simplify the situation, let us consider the case of finite longitudinal size L, when only a single (two-fold degenerate) level lies within the energy interval of about $2E_0$. The level position ξ with regard to the resonance center is determined by the relation $\omega_k - \omega_0/2 = \xi$. Let us represent the function $f_k(t)$ in the form $f_k(t) = |f_k(t)| \exp[i\varphi_k(t)] = N_k(t) \exp[i\varphi_k(t)]$ ($N_k \gg 1$). Then, from Eqs. (15) we find

$$\begin{aligned} \frac{dN_k}{dt} &= -2\gamma N_k + 2E_0 N_k \cos\varphi_k; \\ \frac{d\varphi_k}{dt} &= -2[\xi + \bar{A}N_k + E_0 \sin\varphi_k], \quad \bar{A} = 3A. \end{aligned} \tag{16}$$

The stationary solution of this system has the form

$$N^s_k = \frac{\sqrt{E_0^2 - \gamma^2} \mp \xi}{|\bar{A}|}; \quad \sin\varphi^s_k = \mp\frac{\sqrt{E_0^2 - \gamma^2}}{E_0}. \tag{17}$$

The signs $\mp$ correspond to $\bar{A} \gtrless 0$, respectively. For definiteness, we suppose $E_0 > 0$. The result (17) has an interesting physical origin. In fact, the interaction H_{eff} determines the effective renormalization of the phonon level, leading to $\delta\omega_{k_0} = \bar{A}N_k$ for the case concerned. Accordingly, $\xi \to \bar{\xi} = \xi + \bar{A}N_k$. From Eqs. (17) it straightforwardly follows that $\bar{\xi}_s = \pm\sqrt{E_0^2 - \gamma^2}$. So, at $N_k = N^s_k$ the renormalized level reaches the left or right edge of the parametric energy interval (within the accuracy of the shift due to γ). As a result, the parametric increase of the phonon occupation numbers stops and the phase acquires the constant value. The maximal value of the phonon number equals

$$N^s_{kmax} \approx \frac{E_0}{|\bar{A}|} \gg 1; \quad \gamma, \xi \ll E_0. \tag{18}$$

The phase is $\varphi^s_k \approx \mp\pi/2$ for $\bar{A} \gtrless 0$. The phase φ_k corresponds to the phase correlation of phonon pairs with the opposite momenta. The appearance of the phase φ_k and anomalous correlator f_k is connected, first of all, with the creation of phonon pairs in the course of the evolution as a result of the parametric resonance. The phonon-phonon interaction does not disturb the phase correlation, and the evolution finishes in the stationary condensate of phonon pairs with zero momentum and the common phase φ^s_k.

The character of the temporal evolution, tending asymptotically to the values (17), (18), depends essentially on the relations between the parameters in Eqs. (16). First, let $\gamma = 0$. In this case, Eqs. (16) have the conserved integral of motion

$$H_0 = 2E_0 N_k \sin\varphi_k + \bar{A}N_k^2 + 2\xi N_k. \tag{19}$$

At the initial time moment when $\bar{\xi}_0 = \xi + \bar{A}N_k(0)$ and $\bar{\alpha}_0 = \sqrt{E_0^2 - \bar{\xi}_0^2}$, Eqs.(16) imply that $\cos\varphi_k(0) = \bar{\alpha}_0/E_0$. Bearing in mind that $\sin\varphi_k(0) = -\bar{\xi}_0/E_0$, from the second equation in (16) we obtain $(d\varphi_k/dt)(0) = 0$. As a consequence, the initial (and conserved) value of H_0 is equal to $H_0 \approx -\bar{A}N_k^2(0) \ll E_0$. For an arbitrary time, Eq. (19) can be rewritten as $2E_0 \sin\varphi_k(t) = -[2\xi + \bar{A}N_k(t)] - H_0/N_k(t)$. Substituting this relation into equations (16), one can find

$$\frac{dN_k}{dt} = \pm 2N_k\alpha; \quad \frac{d\varphi_k}{dt} = -\bar{A}N_k - \frac{H_0}{N_k}, \tag{20}$$

$\alpha = [E_0^2 - (\bar{A}N_k/2 + \xi - H_0/2N_k)^2]^{1/2}$. The signs $\pm$ correspond to the regions with $|\varphi_k| \le \pi/2$ and $|\varphi_k| > \pi/2$, respectively. For the region with $|\varphi_k| \le \pi/2$, the solution can be found straightforwardly

$$2t \approx \int_{N_k(0)}^{N_k(t)} \frac{dx}{x[E_0^2 - (\bar{A}x/2 + \xi)^2]^{1/2}}. \tag{21}$$

Here we omitted the small term with $|H_0| \ll E_0$. The upper limit of the integral is equal to the value $N^m_k \approx 2(E_0 \mp \xi)/|\bar{A}|$ at which the denominator vanishes. The divergence at $N_k \to 0$ is a typical manifestation of the parametric resonance which requires the finiteness of the initial field amplitude. This can be achieved by taking zero-point oscillations into account [1]. Owing to these

oscillations, we can put $N_k(0) \sim 1$. The time necessary for the system to achieve the maximal value N_k^m is equal to

$$t_m \approx \frac{1}{2\alpha_0} \ln \Big[\frac{4E_0}{\bar{A}} \cdot \Big(1 - \frac{\xi^2}{E_0^2}\Big)\Big], \qquad (22)$$

We see that the argument of logarithm is much greater than unity at $\xi < E_0$ and $E_0 \gg \bar{A}$. This implies that $t_m \gg 1/2\alpha_0$ where $1/2\alpha_0$ is the characteristic time of the parametric resonance. At $t = t_m$ we have $|\varphi_k| = \pi/2$. As it follows from the second equation in (20), at $t > t_m$ the phase proves to be in the region with $|\varphi_k| > \pi/2$. As a result, in the first equation the sign becomes negative and the phonon number reduces. This is the start of an oscillating behaviour. The joint solution of Eqs. (20) has the form of high anharmonic oscillations of both the phonon number and the phase around their stationary values. At $\xi = 0$ the solution can be written in terms of Jacobian elliptic functions. Averaging the dependence $N_k(t)$ over the large temporal interval $t \gg t_m$, we arrive at value N_k^s.

The character of evolution changes drastically at $\gamma \neq 0$. Namely, a self-averaging arises, leading asymptotically to the stationary value N_k^s. In this case, the integral

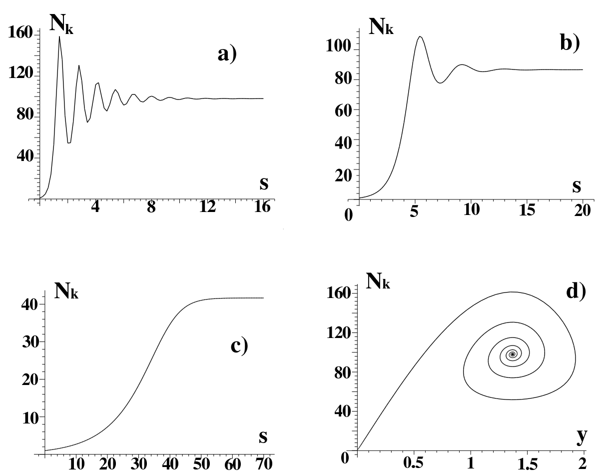

FIG. 1: ! The phonon numbers $N_k(s)$ versus $s = 2\gamma t$. The cases (a), (b), and (c) for $(E_0/\gamma) = 5,\ 2,\ 1.1$. The case (d) plots $N_k(t)$ versus $y(t) = -\varphi_k(t)$ for $(E_0/\gamma) = 5$.

of motion is absent and the solution of Eqs.(16) should be found directly. The damping of the oscillations has the decrement close to γ when $\alpha_0 \gg \gamma$ and $t > t_m$. As γ increases, the arrival time to the stationary state decreases as $1/\gamma$. When γ is close to α_0, the oscillations must disappear completely. In fact, the time necessary for the phonon number to reach the stationary state will be determined by Eq. (22) in which α_0 is replaced by $\alpha_1 = \alpha_0 - \gamma$. The direct numerical simulation of the system (16) demonstrates the picture described. In Figs.1 (a)-(c), the phonon number N_k as a function of $s = 2\gamma t$ is shown at various values of the ratio E_0/γ for the fixed value of the ratio $(E_0/\bar{A}) = 10^2$ and $\xi = 0$. As an illustration, Figure 1 (d) displays the phase portrait in the plane $[N_k, \varphi_k]$ for $(E_0/\gamma) = 5$. Thus the self-consistent temporal evolution of interacting phonons near the parametric resonance results in the formation of the stationary state with the macroscopic phonon number (of the order of $E_0/\bar{A}$) in a single quantum state. The stationary state has one of two energies $\varepsilon_s = \hbar(\omega_0/2 \pm E_0)$. In the spatial sense this is a superposition of two states with the wave vectors k_s and $-k_s$.

The involvement of a coherent combination of phonons with the momenta k_s and $-k_s$ induces the stationary space modulation of the condensate. It is easy to reveal determining the particle density as $\Delta n =< \hat{\chi}'^{+}(\rho, z)\hat{\chi}'(\rho, z) >$, where $\hat{\chi}'$ has the form (8). Rewriting $\hat{\chi}'$ in terms of phonon operators and integrating over $d^2\rho$, we arrive at the expression for the modulation of the 1D atomic density

$$\frac{\delta n(z)}{n^{(1)}} = \frac{2N_k^s}{N}(u_{k_s}^2 + v_{k_s}^2)\cos 2k_s z \approx \frac{2\mu}{\varepsilon_s} \cdot \frac{N_k^s}{N} \cdot \cos 2k_s z, \quad (23)$$

where $n^{(1)} = N/L$. We suppose that $|\delta n|/n^{(1)} \ll 1$.

Let us make here some estimations. As an illustration, we consider a quasi-1D gas of sodium atoms. The quasi-1D situation occurs under condition $\mu < \omega_\perp$. Here $\mu \sim (an^{(1)})\hbar\omega_\perp$ (see [8]), a being the 3D scattering length. For both $n^{(1)} = 10^6 cm^{-1}$ and $L \approx 10^{-2}cm$, we have $an^{(1)} \approx 0.2$ and $N \approx 10^4$. Assuming $(\omega_\perp/\omega_0) = 5$, we find

$$N_k^s \approx 10^4 \cdot \frac{|\delta\omega_\perp|}{\omega_\perp}; \quad \frac{|\delta n|}{n^{(1)}} \approx \frac{3|\delta\omega_\perp|}{\omega_\perp}.$$

The above analysis is performed for the case when a single level lies within the energy interval $2E_0$. This assumption imposes certain restrictions on the ratio $E_0/\Delta\omega$ where $\Delta\omega = \pi c/L$ is the spacing between the neighbor longitudinal levels, c being the sound velocity. Combining definition of E_0 (11) and value $\omega_\perp = 10^4 1/sec$, we find $(E_0/\Delta\omega) \approx 10|\delta\omega_\perp|/\omega_\perp)$. These estimations demonstrate that at $|\delta\omega_\perp|/\omega_\perp) \sim 10^{-2}$ the conditions $|\delta n|/n^{(1)} \ll 1$, $E_0/\Delta\omega \ll 1$ are satisfied and value N_k^s proves to be about 10^2. In reality, we may increase the ratio $(|\delta\omega_\perp|/\omega_\perp)$. As a consequence, the value N_k^s should be within interval $10^2 - 10^3$. The parameter γ/E_0 essential for the temporal evolution may be changed varying the ratio $|\delta\omega_\perp|/\omega_\perp$. Thus, we can conclude that the phonon condensation can be realized for a quite realistic set of parameters. It should be noted that, for the finite but sufficiently low temperatures when the initial number of phonons with $\omega_k \sim \omega_0$ is limited, the results obtained remain valid.

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[1] Yu.Kagan and L.A.Maksimov, Phys.Rev.A, **64**, 053610 (2001); Laser Physics, v.12, p.153 (2002).

[2] F.Chevy, V.Bretin, P.Rosenbusch, K.W.Madison, and J.Dalibard, Phys.Rev.Lett., **88**, 250402 (2002).

[3] Yu.Kagan and L.A.Maksimov,Phys.Lett.A,**317**,477(2003).

[4] Yu.Kagan, E.L.Surkov, and G.V.Schlyapnikov, Phys. Rev. A, **54**, R1753 (1998).

[5] E.Zaremba, Phys.Rev.A, **57**, 518 (1998); S.Stringari, Phys. Rev. A, **58**, 2385 (1998).

[6] Fr.Dalfovo, St.Giorgini, L.P.Pitaevskii, and S.Stringari, Rev.Mod.Phys., **71**, 463 (1999).

[7] E.M.Lifshitz and L.P.Pitaevskii, Statistical Physics (Pergamon, Oxford, 1980), Part 2.

[8] M.Olshanii, Phys.Rev.Lett., **81**, 938 (1998).